PSA in Zero Gravity
Artist Concept

B Background

Scientists at NASA are building a robot called the Personal Satellite Assistant, or PSA. The PSA's mission is to keep the astronauts safe and to assist them with their chores on space-based vehicles such as the International Space Station (ISS), a Crew Exploration Vehicle, or even on Mars. This small round robot will float in microgravity and move autonomously (without direction from people). It will keep track of the astronauts' schedules, monitor supplies, assist with scientific experiments, communicate with Mission Control, and help keep the astronauts safe by monitoring the air composition and temperature.

NASA engineers have created a model of the PSA with a diameter of 30.5 centimeters (12 inches). The engineers' goal is to build a PSA with a 20-centimeter (8-inch) diameter, because it will be safer and will require less power to move around.

I Introduction

In this lesson, we assume that the computer inside the PSA is in the shape of a rectangular prism. Students use calculations, visualization, and logical reasoning to discover that the surface area and volume of a rectangular prism change disproportionately when the linear dimensions of the prism change. This lesson will also introduce ratios as a method of comparing physical quantities.

M Main Concept

The surface area and volume of a rectangular prism change disproportionately when the linear dimensions change.
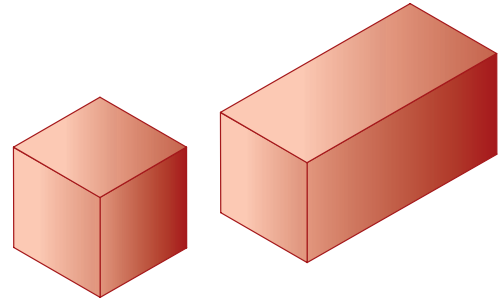
N NASA Relevance

NASA scientists and engineers working on the PSA project need to reduce the volume and mass of the PSA because of the high cost of transportation for leaving our home planet, limited space in space-based vehicles, and for safety reasons. Even though they use computer programs to design the PSA and its components, a basic understanding of volume and surface area is essential in order to design the PSA and the shape of the components that go inside it.

P Prerequisite Skills

Students should be able to:
- Explain that a flat rectangular prism has a greater surface area than a thick rectangular prism of equal volume (see Lesson 1: Surface Area and Linear Dimensions of a Rectangular Prism).
- Calculate and interpret ratios.
- Multiply and divide decimals.

I Instructional Objectives

During this lesson, students will:
1. Use ratios to compare physical quantities.
2. Explain that the surface area and volume of solids change disproportionately when their linear dimensions change.
3. Make a recommendation as to which dimension of a computer should be doubled in order to double its volume and maximize its surface area.

| NATIONAL EDUCATION STANDARDS		
Fully Met	**Partially Met**	**Addressed**
NCTM (6-8) *Geometry #1* *Measurement #2* **2061: 9B (6-8) #2**	**NCTM (6-8)** *Numbers and Operations #1* *Geometry #4*	**NCTM (6-8)** *Measurement #1* **2061: 2B (6-8) #1**

(M) **Major Concepts**
- Mathematical statements can be used to describe how one quantity changes when another changes.
- Ratios can be used to understand mathematical relationships or to compare changes.
- As the dimensions of a rectangular prism are increased, the volume increases more than the surface area.

(M) **Materials and Equipment**
- 4 wooden or plastic rectangular prisms of equal size per group of 2 or 3 students
- Rulers or tape measures for each group
- Double-sided tape for each group (optional)
- 1 copy of the Student Handout sheet for each student or group of students
- Calculators (optional)

(T) **Time for Activity**
1–2 class periods
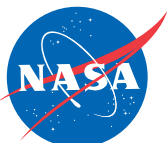
LESSON   Engage

Review with students what they learned from Lesson 1 about the relationship between surface area and the dimensions of a rectangular prism. Ask students how the observation of this relationship helped them to come up with a solution for the dimensions of the PSA's computer. Ask them how it helped them to make a recommendation on the computer stacking problem.

Point out that understanding mathematical relationships can be very helpful in solving many problems. Tell students that NASA's PSA robot will require many different components that are not only shaped like rectangular prisms but also cylinders. To help NASA make other decisions about the dimensions for these components, they will be looking more closely at the relationships between surface area and volume of these shapes.

Introduce the problem that students will be exploring in this lesson. Tell students that engineers have been asked to double the power of a computer by doubling its volume. They need to see what will happen to the surface area when they do this, as it is important to have the maximum surface area to release heat. They will need to recommend which dimension of the computer should be doubled.

Ask students what math relationships might help them to solve this problem. Tell students that in order to solve this problem they will be examining how changes in the length, width, and height of a rectangular prism affect its volume and surface area.

Computing in Space
Astronaut Thomas B. Jones

Review with students that a ratio is a comparison of how much bigger or smaller one measurement is than another. If students have not been introduced to ratios, it will probably be useful to show some examples in class.
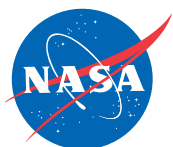
Show students two rectangular prisms of equal size. Confirm with the students that the rectangular prisms have equal volume and surface area.

Using double-sided tape, affix the two rectangular prisms together to form one large rectangular prism.

Ask students how to determine how much more volume the large prism has than the individual small prisms.

Ask students how to determine how much more surface area the large prism has than the individual small prisms.

Again, ask the students how much more surface area they think the large prism has than the small ones.

Review formulas for calculating the surface area and volume of rectangular prisms:

Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)
Volume = length x width x height

Explore

Students may use calculators if they choose.

Distribute four rectangular prisms of equal size to each group of students. Using rulers or tape measures, ask students to measure the length, width, and height of an individual rectangular prism. Instruct the students to calculate the surface area and volume of an individual prism. Students should record their answers on the Student Handout sheet.

Using double-sided tape, ask students to attach two rectangular prisms together. Students can attach the prisms to increase any of the linear dimensions (length, width, or height). Tell students to measure the length, width, and height of the new prism. Instruct students to calculate the surface area and volume of the larger rectangular prism. Ask students to compare the surface area and volume of the large rectangular prism to the surface area and volume of an individual rectangular prism, using ratios. Students should record their answers on the Student Handout sheet.

Ask students to repeat the above procedure by adding a third rectangular prism and then a fourth rectangular prism. Students should only change one linear dimension. So, if students increased the length of the prism in the first step, they should continue to increase the length of the rectangular prism in the remaining steps. Ask students to calculate the surface areas and volumes for each new rectangular prism.

Explain

Ask students how the volume of the largest rectangular prism compares to the volume of an individual prism. It will help to compare the ratios obtained by student groups who changed the lengths to groups who changed the widths to groups who changed the heights of their rectangular prisms. If none of the student groups changed one of the dimensions, ask the students to speculate how they think the volume would change if that dimension were increased.

Repeat these questions for surface area.
Ask students: "Why doesn't the surface area increase as much as the volume when you change the dimensions of the rectangular prism?"
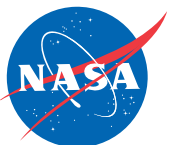
Listen to all of the students' responses. The surface area does not increase as much as the volume because when rectangular prisms are connected, the areas of the connecting faces are removed from the total surface area. So, while the volumes simply double each time, the surface area does not increase as much.

Ask students to count the number of faces that are visible in the largest rectangular prism. Ask them to count the number of faces that are not visible in the prism.

Ask: "How do the faces that are not visible affect the surface area of the prism?"

If students do not realize that the invisible faces make the surface area change less than the volume, ask them to calculate the surface area of four individual rectangular prisms. Tell the students to add the surface areas of four individual prisms to determine a total surface area. Ask students to compare this total surface area to the surface area of the largest rectangular prism they created during the activity.

For the largest rectangular prism, ask students to calculate the areas of the invisible faces. Instruct students to compare the areas of invisible faces for high, long, and wide prisms.

Ask students if there is any connection between the area of the concealed faces and the total area of the large rectangular prism.

Extend

Engineers had a computer of dimensions 6 cm x 4 cm x 1 cm. The computer company decided they wanted the engineers to design a computer with twice the power. Engineers decided they would simply double the volume of the computer, using twice as many components to double its capabilities. What will happen to the surface area to volume ratio when doubling the volume? Which dimension would you recommend they double in order to maximize the surface area? Why?

Evaluate

As a class, create an assessment rubric for this activity. Suggested criteria for the rubric include:
- Accurate measurements of length, width, and height of rectangular prisms.
- Accurate calculations of surface area and volume.
- Appropriate interpretations of ratios.
- Correct assessment of the relationships between volume, surface area, and linear dimensions of a rectangular prism.
- Clear oral reasoning as to why the surface area of a rectangular prism changes at a different rate than its volume.
- Clear written presentation of results.
- Clear oral presentation of results.

Use the rubric to assess students' solutions and ensure they have mastered the major concepts and math skills.
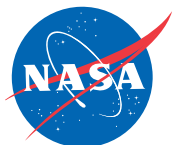
Student Handout

Formulas

Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)
Volume = length x width x height

Increasing Prism Dimension

1. Use centimeters for your measurements.

One Individual Rectangular Prism

Length	Width	Height	Volume	Surface Area

Two Attached Rectangular Prisms

Length	Width	Height	Volume	Surface Area
Ratio of Volume to the Volume of One Individual Rectangular Prism				
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				

Three Attached Rectangular Prisms

Length	Width	Height	Volume	Surface Area
Ratio of Volume to the Volume of One Individual Rectangular Prism				
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				

Four Attached Rectangular Prisms

Length	Width	Height	Volume	Surface Area
Ratio of Volume to the Volume of One Individual Rectangular Prism				
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				

2. Does the surface area of a rectangular prism change more or less than the volume as it gets bigger?

_____

_____

3. Why do you think this is?

_____

_____

_____

Doubling Computer Volume

1. Engineers had a computer of dimensions 6 cm x 4 cm x 1 cm. The computer company decided they wanted the engineers to design a computer with twice the power. Engineers decided they would simply double the volume of the computer, using twice as many components to double its capabilities. What will happen to the surface area to volume ratio when doubling the volume? Which dimension would you recommend they double in order to maximize the surface area? Why?

_____

_____

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_____

Answer Key

Formulas

Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)

Volume = length x width x height

Increasing Prism Dimension

Example:

One Individual Rectangular Prism

Length	Width	Height	Volume	Surface Area
10 cm	5 cm	2 cm	100 cm³	160 cm²

Two Attached Rectangular Prisms (prism is added to increase length dimension)

Length	Width	Height	Volume	Surface Area
20 cm	5 cm	2 cm	200 cm³	300 cm²
Ratio of Volume to the Volume of One Individual Rectangular Prism				200/100 = 2
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				300/160 = 1.875

Three Attached Rectangular Prisms (prism is added to increase length dimension)

Length	Width	Height	Volume	Surface Area
30 cm	5 cm	2 cm	300 cm³	440 cm²
Ratio of Volume to the Volume of One Individual Rectangular Prism				300/100 = 3
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				440/160 = 2.75

Four Attached Rectangular Prisms (prism is added to increase length dimension)

Length	Width	Height	Volume	Surface Area
40 cm	5 cm	2 m	400 cm^3	580 cm^2
Ratio of Volume to the Volume of One Individual Rectangular Prism				400/100 = 4
Ratio of Surface Area to the Surface Area of One Individual Rectangular Prism				580/160 = 3.625

The ratio of surface areas will always be less than the ratio of volumes. When two rectangular prisms are combined into one, two faces are concealed and are thus not added into the total surface area. In contrast, none of the volume is concealed when two prisms are combined.

Doubling Computer Volume

Engineers had a computer of dimensions 6 cm x 4 cm x 1 cm. The computer company decided they wanted the engineers to design a computer with twice the power. Engineers decided they would simply double the volume of the computer, using twice as many components to double its capabilities. What will happen to the surface area to volume ratio when doubling the volume? Which dimension would you recommend they double in order to maximize the surface area? Why?

As the volume is doubled, the surface area will increase, but not as much. Students should recommend that the largest dimension or length (6 cm) be doubled, to maximize the surface area. This will cause the smallest faces (of 4 cm x 1 cm) to be together, so that the surface area is maximized. The new volume will be 48 cm^3 and the surface area would be 128 cm^2. (The surface area when doubling the width is 124 cm^2 and the surface area when doubling the height is 88 cm^2).

Sample Scoring Tool

4
- Calculations are correct and clearly presented.
- Students accurately interpret ratios.
- Reasoning is logical and clear explanations are provided.
- Oral and written presentations are clear.

3
- Most calculations are correct and attempts are made to present them clearly.
- Students draw appropriate conclusions from ratios.
- Attempts are made to reason logically and provide clear explanations.
- Attempts are made to provide clear oral and written presentations.

2
- Some calculations are correct and attempts moderately clear.
- Students have difficulty interpreting ratios.
- Explanations demonstrate limited logical bases.
- Oral and written presentation skills need improvement.

1
- Few calculations are correct and attempts are unclear.
- Students do not demonstrate adequate understanding of ratios.
- Explanations do not demonstrate understanding of lesson content.
- Oral and written presentations do not effectively express results or reasoning